Combined Mechanisms of Collapse of Discrete Single-Layer Spherical Domes

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Abstract

This paper presents investigation of combined mechanisms of collapse of discrete single-layer domes. Main factors influencing sustainable behavior are: morphology of the grid configuration, geometry and material characteristics of the structural elements, type of joints and the supporting conditions. On this basis, representative configuration of discrete spherical dome for structural analysis is selected. The numerical solution is performed by software based on the Finite Element Method. Several load cases are considered. These are consistent with the surfaces of influence that have been determined. In order to define the critical value of the load parameter, the loading is increased incrementally and iterative procedure is used in a given load step in order to exclude the structural elements which have reached their limit capacity. Thus the boundary equilibrium of the system is defined. The study reveals geometric nonlinearities by accounting for P- Δ effects, i.e. the influence of normal forces on the stiffness of the system. It is assumed that the displacements increase after exclusion of the elements which have reached their limit capacity. The equilibrium conditions are defined for the deformed shape of the structure. Results are presented in graphical and tabular form.

Keywords

Single-Layer Domes, Mechanisms of Destruction, Loss of Strength and Stability

Introduction

The study of the sustainable behavior of discrete domes is integral part of their analysis and design. Scientists all over the world have conducted numerous studies and experiments in order to ensure their stability and reliability. One of the reasons for such extensive research is the large number of cases of domes which failed due to loss of stability under loading considerably smaller than the critical loading of the system. The main factors influencing sustainable behavior of discrete domes are: morphology of the grid, geometry and material characteristics of structural elements, type of joints and the supporting conditions.

Gioncu [8, 9] describes in detail the status of the research of the sustainable behavior of single layer lattice domes. Some important studies in this area are made by Sumec [10]. The extensive use of discrete computational models for the calculation of plane and space structures is closely related to the rapid development of effective software programs. Gioncu identifies four main factors that have particular importance on the sustainable behavior:

- form of the grid configuration;
- structural composition of grid configuration: triangular or quadrilateral;
- geometrical and physical characteristics of the individual structural element;
- type of connections: welded, bolted or special mechanical joint.

The main task of the study of the sustainability of lattice domes is to determine the critical load parameter. Three types of analysis theories can be applied:

1) theory of first order: equilibrium equations are defined for the undeformed system;

2) theory of second order: equilibrium equations are defined on a deformed system, but the displacements are small compared to the dimensions of the system.

3) theory of third order: equilibrium equations are defined for deformed system at any magnitude of the displacements.

The studies related to the problem of stability of structures using FEM are given in detail in [1, 2, 5, 6]. A characteristic of the calculation is the so-called geometric stiffness matrix that takes into account the influence of the normal forces on the stiffness of the system.

For the calculation of critical loads, based on second

and third order theory the loads have to be treated as a process because of the non-linear behavior of the system.

Gioncu and Balut [8] illustrate some iterative methods to obtain solutions and develop the thesis that after determination of the bifurcation load, an investigation according to second and third order theory is required in order to specify whether the system is vulnerable to initial imperfections. There are several steps in the study of sustainability. These steps are:

- Determine the type of nonlinear analysis to be applied: geometric nonlinear elastic analysis and geometrical and physical nonlinearity with elasto-plastic analysis;
- Select the physical model: a discrete system or an equivalent continuous model;
- Select a computing model and computational procedure for nonlinear pre-critical study and after critical behavior;
- Accounting for factors affecting bearing capacity: density of grid structure, geometric and mechanical tolerances, plastic deformation, stiffness of the connections, distribution.

Detailed analyses indicate the main unsustainable forms of single layer lattice domes are as follows:

1) Instability of a single element - Individual element is unstable or excluded after reaching its limit capacity, but the structure continues to work. Geometric deviations are the main reason for loss of stability of the elements. This raises the question whether to take into account the influence of such elements on the behavior of the entire structure. The answer depends on the critical behavior of the element. In most cases, all other structural elements connected to certain unstable element have a stabilizing effect on it. It is essential to ensure continuity of the structural system especially of those elements belonging to ring and radial beams of the cylindrical shells.

2) Local loss of stability:

a/ Unacceptable deviation of a single node or group of nodes – this is when significant nodal displacement is observed referred to the original middle surface of the dome structure. The phenomenon is known as "pitting" in spherical shells. This requires examination of the after critical behavior of the system. Many researchers are on the opinion that this phenomenon leads to the failure of the structure. Pitting occurs when a node is considerably heavier compared to its neighboring nodes, or when a group of nodes are equally loaded, but one of them deviates significantly from the initial geometry. Such behavior is referred to as sensitivity of the structure to nodal imperfections.

b/ Unacceptable linear displacement of a node – this is observed when all nodes in a given direction (meridian or parallel) are loaded with significantly larger forces compared to nodes outside the given direction. This behavior is similar to the one in a/.

c/ General instability - stretches over a large area of the structure consisting of several elements and nodes.

d/ Combined instability – such forms of instability are observed when the value of the critical load corresponds to two separate unstable forms (usually nodal and lattice).

General characteristic feature of the single layer oblique lattice domes is the nonlinear behavior with progressive decreasing stiffness. Large numbers of scientists all over the world are dealing with problems of stability of lattice domes. Guidelines for their research related to the sustainable behavior under static and dynamic loading are defined in the context of physical and geometric nonlinearity.

Morris [12] shows that the critical behavior gives information about the sensitivity to geometric deviations of the system. Ueki, Kato, Kubodera and Mukaiyama [13] perform parametric study of sustainable behavior of single layer lattice dome. Murakami and Heki [11] discuss the stability of lattice dome under the action of vertical load. Critical loads and corresponding forms of loss of stability are determined by FEM program. Bori and Chiostrini [15] study post-critical behavior of lattice dome with a radius $R=30m$ and $H=3m$ for primary variations in the elements. They develop a numerical method based on fundamental relationships in FEM, in which the effect of these variations is expressed by additional (fictitious) nodal forces. They assume that the value of the deviation increases proportionally to the radius of inertia of the element. Sumec [10] determines the critical load for sloping lattice dome with the analogy of a continuous shell. Depending on the geometry and density of the grid, the author has developed a method in which by means of charts and analytical expressions the critical load can be determined. The influence of the primary deviation is reflected by proofreading coefficients. Holzer and Tissaoui [16] give numerical solution by the FEM for stability studies in three cases of snow loads.

The results show considerable differences in the behavior of the domes when applying linear and nonlinear analysis. The critical load for the dome with geometric variations is much smaller than the one with ideal geometry. Ikarashi and Kato [17] investigate dome with three models of connections: hinge, rigid and semi rigid connection.

Judging from current analyses, it can be concluded that the investigation of the behavior of spatial trusses is a major challenge for civil engineers, both practitioners and researchers. Considerable interest has been shown by scientists and researchers all over the world in this area, as evidenced by the large number of publications and articles in scientific journals. Most programming systems have possibilities for more detailed analysis of space systems using linear and nonlinear solution analysis in terms of geometric nonlinearity of the elements of structure and physical nonlinearity of the material.

Investigation of the Discrete Single-Layer Spherical Dome in a State of Equilibrium

This article investigates a typical discrete spherical dome as the one shown in Fig. 1. The loss of stability of this dome is connected to a hidden form of deformation of the structural elements when the critical load is reached. Investigation of the boundary state of stress and strain of the structure indicates that the following features have to be considered:

- Due to prevailing compression in the dome elements and their decisive role in ensuring stability and rigidity, it is assumed that the mechanisms of destruction will occur by exclusion of the bars that are most heavily loaded in compression.
- Due to the large number of elements and nodes it is impractical (or even impossible) to search for independent mechanisms of failure under the action parametrically increasing forces.

The following dimensions and assumptions are considered for the dome under investigation:

- base ring diameter *D=30m*;
- diameter of key ring d_k *=1,65m*;
- cyclic angle at the base $\theta = 15°$;
- central semi angle α*=50°* and corresponding height of the dome *H=7m*;
- number of inner rings *n=7*;
- rigid connection between the elements of the grid structure and the base ring;

rigid connection between the elements of the grid structure.

Pipe sections *Ø83x4* with cross-section area $A=9,93cm^2$, inertial moment $I=77,64cm^4$ and inertial radius *i=2,8cm* are assumed for the structural elements. Steel grade *S235JRH* according to EN 10219-2 with characteristic strength *Ry=235MPa* is used.

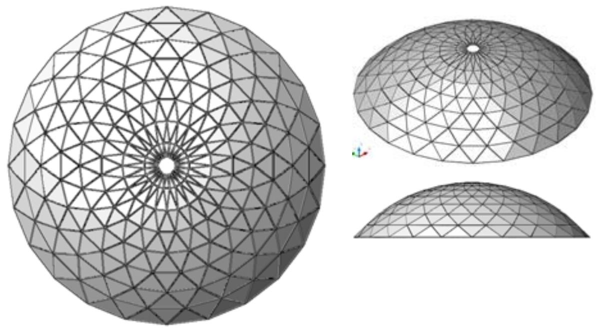


FIG. 1 DISCRETE SPHERICAL DOME UNDER INVESTIGATION

The limit compression load bearing capacity of the diagonal and the ring elements is calculated by considering their slenderness and the results are shown in Table 1 and Table 2.

TABLE 1 LIMIT LOAD BEARING CAPACITY OF THE DIAGONAL ELEMENTS FOR COMPRESSION

	L_i	A	i	λ
first level	283,45	9,93	2,8	101,2321
second level	270,55	9,93	2,8	96,6250
third level	257,48	9,93	2,8	91,9571
fourth level	244,87	9,93	2,8	87,4536
fifth level	233,43	9,93	2,8	83,3679
sixth level	223,93	9,93	2,8	79,9750
seventh level	217,08	9,93	2,8	77,5286
eighth level	132,32	9,93	2,8	47,2571
	$\bar{\lambda}$	φ	R_y	$N_{cr,i}$
first level	3,4192	0,5409	23,5	119,9081
second level	3,2635	0,5717	23,5	126,7467
third level	3,1059	0,6040	23,5	133,9074
fourth level	2,9538	0,6362	23,5	141,0374
fifth level	2,8158	0,6662	23,5	147,694
sixth level	2,7012	0,6918	23,5	153,3577
seventh level	2,6186	0,7105	23,5	157,5181
eighth level	1,5961	0,8650	23,5	191,7595

Limit load bearing capacity of the elements in tension is *221,69kN*. The self weight of the structure is considered. The computational solution is performed with software based on the Finite Element Method. Various load cases are considered – local loads, symmetric and anti-symmetric loads. The load pattern is consistent with the influence surfaces defined in [7].

TABLE 2 LIMIT LOAD BEARING CAPACITY OF THE RING ELEMENTS

	L_i	A	i	λ
first level	353,48	9,93	2,8	126,2429
second level	311,18	9,93	2,8	111,1357
third level	265,18	9,93	2,8	94,7071
fourth level	216,03	9,93	2,8	77,1536
fifth level	164,31	9,93	2,8	58,6821
sixth level	110,64	9,93	2,8	39,5143
seventh level	55,65	9,93	2,8	19,8750
eighth level	21,54	9,93	2,8	7,6929
	$\bar{\lambda}$	φ	R_y	$N_{cr,i}$
first level	4,2639	0,3913	23,5	86,7537
second level	3,7537	0,4781	23,5	105,9781
third level	3,1988	0,5849	23,5	129,6605
fourth level	2,6059	0,7134	23,5	158,1615
fifth level	1,9820	0,8648	23,5	191,7191
sixth level	1,3346	1,0393	23,5	221,6873
seventh level	0,6713	1,2366	23,5	221,6873
eighth level	0,2598	0,8650	23,5	221,6873

In order to determine the critical value of the load parameter incrementally increasing loading is applied with successive approximations for calculation of the boundary equilibrium of the system in each load-step. At a given stage of loading when an element reaches its limit bearing capacity it is excluded from work and in the next step of calculation its contribution is not considered. This is repeated until the equilibrium of the system is reached for this stage of loading without the "excluded" elements. The study was conducted in terms of geometric nonlinearity introduced by accounting for P-Δ effects, i.e. influence of normal forces on the stiffness of the system. It is assumed that the displacements increase after exclusion of elements after reaching their capacity. The equilibrium conditions are defined for the deformed shape of the structure.

Analysis of the Results

First Load Case

The first load case is the local load shown in Fig. 2.

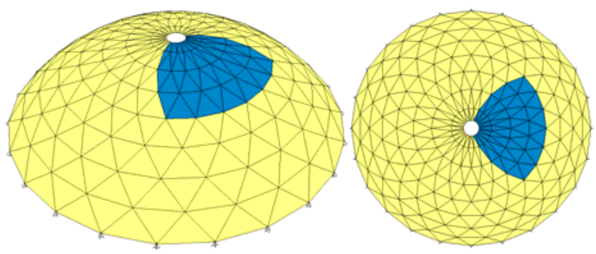


FIG. 2 LOAD CASE 1 - LOCAL LOAD

It has initial value $q_1=1kN/m^2$ and is increased with equal step. When the load reaches $q_8=8kN/m^2$ two elements reach their capacity and are excluded from the next calculation stage (see Fig. 3 and Fig. 5).

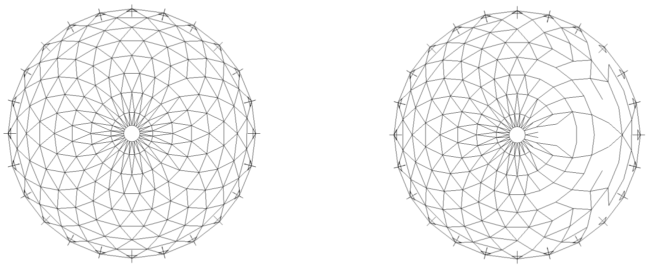


FIG. 3 FIRST AND FIFTH STAGE OF EXCLUSION OF ELEMENTS

In the next calculation steps progressive exclusion of 8, 33, 47 and in the end 12 elements occurs. This process is illustrated in Fig. 5. When the load is $q_8=8kN/m^2$ total of 102 elements reach their capacity. Deformed scheme of the last stage of exclusion is illustrated in Fig. 4.

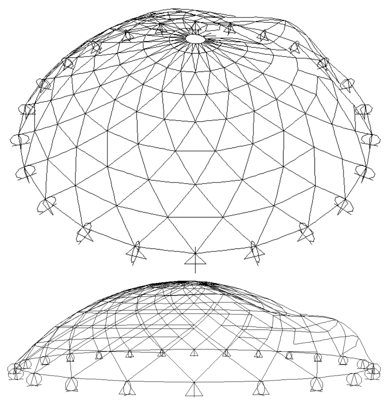


FIG. 4 LAST STAGE (FIFTH STAGE) OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 1

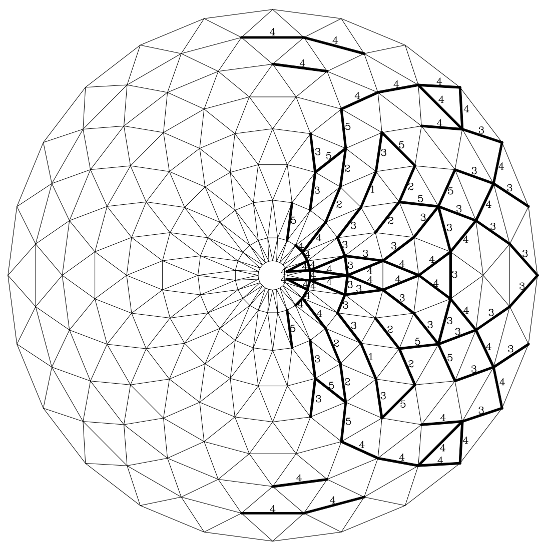


FIG. 5 NUMBER AND SEQUENCE OF BARS REACHING THEIR CAPACITY FOR LOAD CASE 1

Second Load Case

In the second case, the load is applied in anti-symmetric pattern (ratio 2:1) with respect to one of the axis (for example, axis Y) of the dome, see Fig. 6. The first elements reach their capacity at $q_4=4kN/m^2$. The

sequence of exclusion of elements at this stage of loading is shown in Fig. 7 and Fig. 8. First to be excluded are elements number "1", which leads to overloading of elements "2". They are excluded from the next stage, and this process continues until the structure no longer fulfills its operational purpose. This promptly leads to overloading of consecutive groups of elements, increasing the displacement and destruction of the dome. The last stage of exclusion of elements and the corresponding deformed scheme is shown in Fig. 9. The process of successive exclusion is progressive, resembling a "domino effect." When load $q_4=4kN/m^2$ is reached totally 94 elements (in 1st stage - 4, in 2d stage - 16, in 3d stage – 74) are excluded.

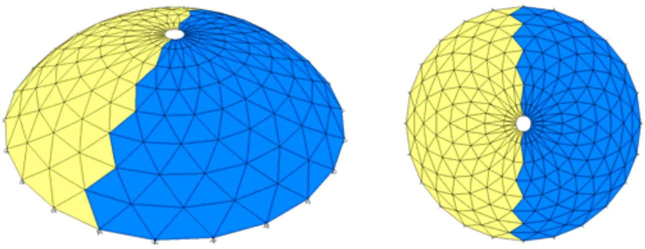


FIG. 6 ANTI-SYMMETRICAL LOAD CASE 2

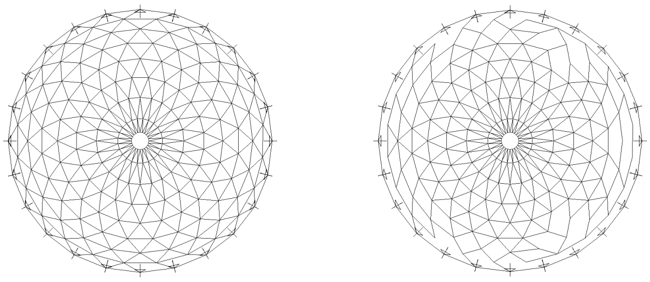


FIG. 7 I-ST STAGE AND III-RD STAGE OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 2

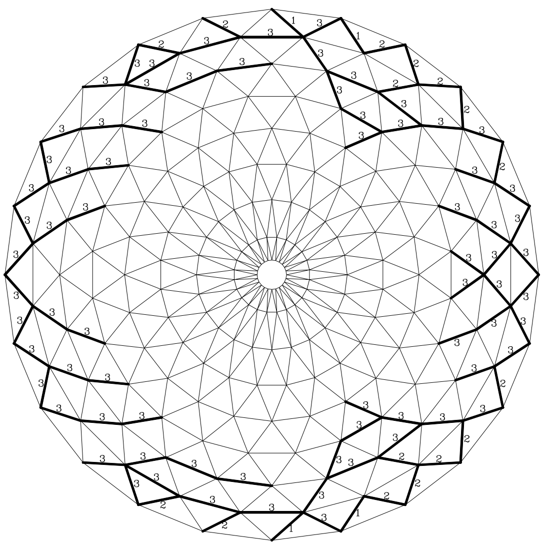


FIG. 8 NUMBER AND SEQUENCE OF BARS REACHING THEIR CAPACITY FOR THE SECOND ANTI-SYMMETRICAL LOAD CASE

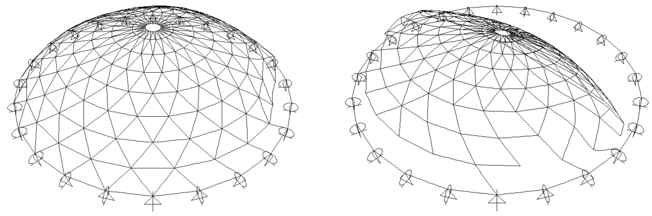


FIG. 9 SECOND AND LAST STAGE (THIRD STAGE) OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 2

Third Load Case

In this case the load is localized between the sixth, seventh and eighth level, see Fig. 10. Step load is applied and the elements start reaching their capacity at $13kN/m^2$.

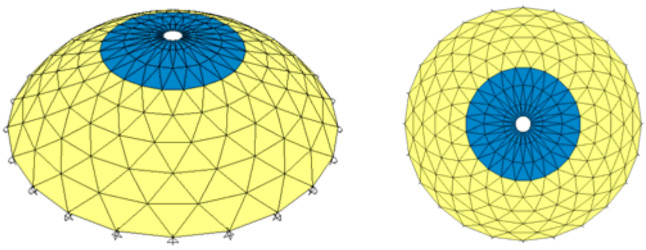


FIG. 10 LOCAL LOAD CASE 3

In the first stage bars from the sixth and seventh inner ring (48 in number) are excluded because of reaching their capacity in compression, then at the second stage elements are excluded from the fourth inner ring (in tension), then from the fifth and eighth inner ring (in compression). After these two stages static scheme of the elements changes and they begin to work in bending. In the third stage they are overloaded by the bending moments occurring in them (144 in number - the diagonal elements in the loaded area).

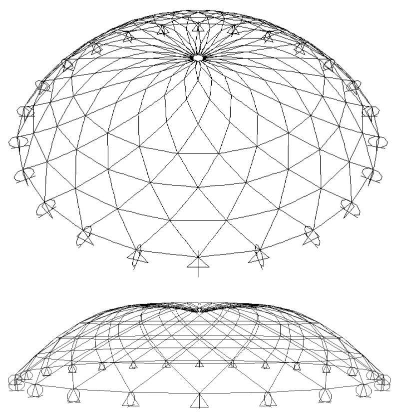


FIG. 11 SECOND STAGE OF THE EXCLUSION OF BARS

When the load is $q_{13} = 13kN/m^2$ total of 264 elements (in 1st stage - 48, in 2d stage – 72, in 3d stage – 144) are excluded from the system. This process is illustrated in Fig. 11, Fig. 12 and Fig. 13.

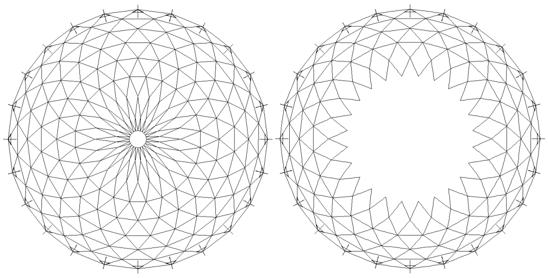


FIG. 12 I-ST STAGE AND III-RD STAGE OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 3

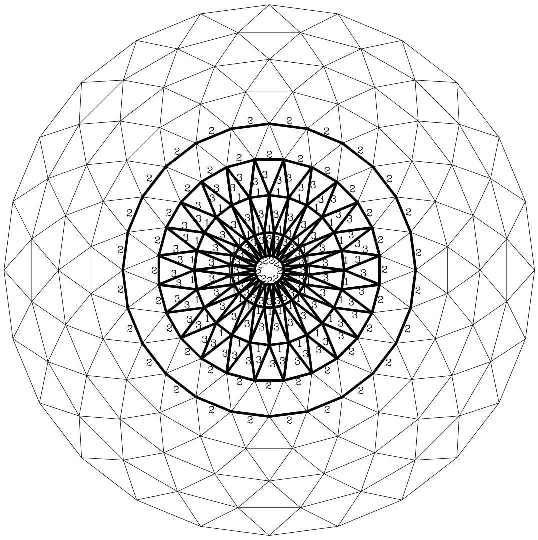


FIG. 13 NUMBER AND SEQUENCE OF BARS REACHING THEIR BEARING CAPACITY FOR LOAD CASE 3

Fourth Load Case

In this case the load is localized in seventh and eighth level as shown in Fig. 14.

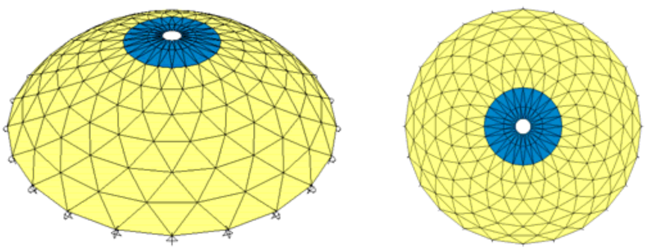


FIG. 14 LOAD CASE 4

At $14kN/m^2$ the elements start reaching their capacity. First elements from the eighth inner ring working under compression are excluded (24 in number), then in the second stage elements of the seventh inner ring (in compression) are excluded, and in the third stage elements of the fifth inner ring (in tension) and sixth inner ring (in compression) reach their capacity.

After these three stages the static scheme of the elements changes and they begin to work in bending as in the previous load case. In the fourth stage the elements are overloaded bending moments (264 in number - the diagonal elements in loaded area). Totally 360 elements (in 1st stage - 24, in 2d stage – 24, in 3d stage – 48, in 4th stage – 264) are excluded at $q_{14}=14kN/m^2$. This process is illustrated in Fig. 15, Fig. 16 and Fig. 17.

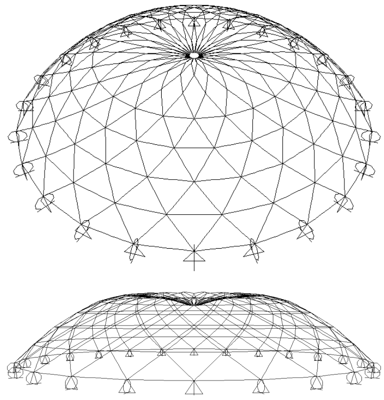


FIG. 15 THIRD STAGE OF THE EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 4

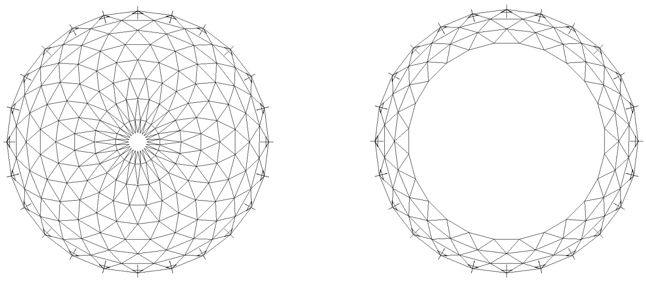


FIG. 16 I-ST STAGE AND IV-TH STAGE OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 4

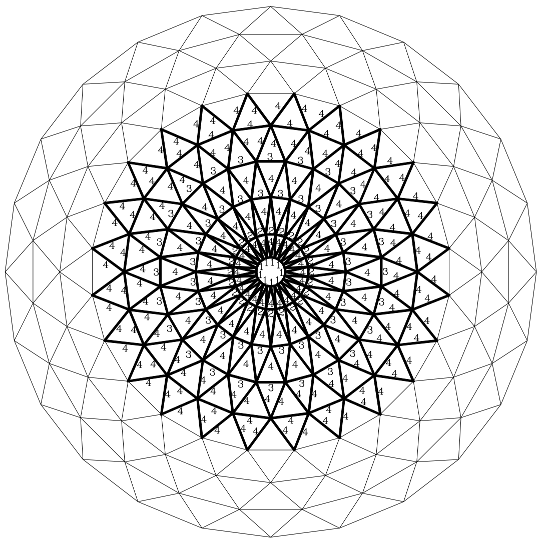


FIG. 17 NUMBER AND SEQUENCE OF ELEMENTS REACHING BEARING CAPACITY FOR LOAD CASE 4

Fifth Load Case

This local load case is shown in Fig. 18.

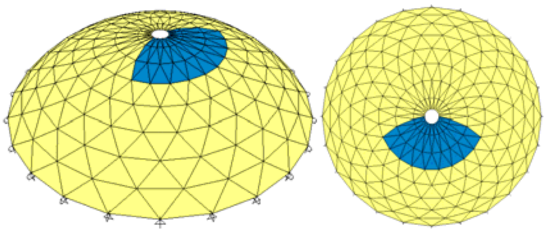


FIG. 18 LOAD CASE 5

When the value of the load is $q_{10}=9kN/m^2$, two elements lose stability and are excluded from the next calculation stage. Consecutive exclusion of 9 elements and then another 15, 78 and 38 occurs reaching total number of 142 (see Fig. 19 and Fig. 20).

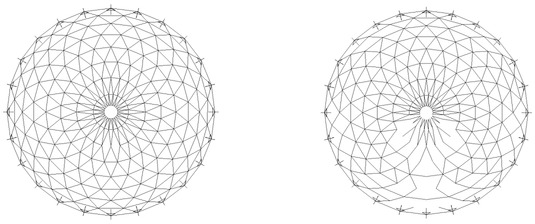


FIG. 19 I-ST STAGE AND IV-TH STAGE OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 5

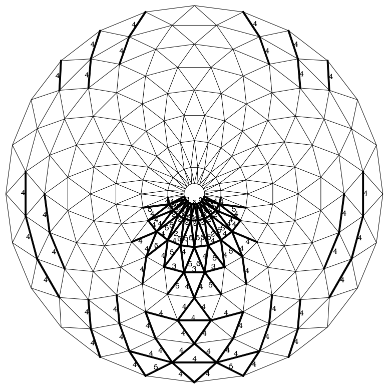


FIG. 20 NUMBER AND SEQUENCE OF ELEMENTS REACHING BEARING CAPACITY FOR LOAD CASE 5

The last stage of the exclusion of bars and its corresponding deformed scheme is shown in Fig. 21.

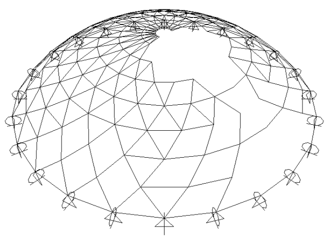


FIG. 21 FINAL STAGE (FIFTH STAGE) OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 5

Sixth Load Case

In the sixth case, the load is applied in anti-symmetric pattern (in ratio 3:1) with respect to one of the axis (for example, axis Y) of the dome as illustrated in Fig. 22. At loading $q_4=4kN/m^2$ the elements start reaching their capacity. First to lose stability are the elements with number "1", which leads to overloading of the elements "2" (see Fig. 23 and Fig. 24), which are excluded from the next stage. This continues until the structure no longer fulfills its operational purpose.

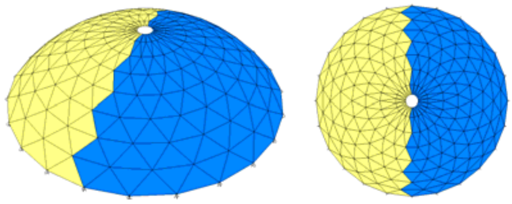


FIG. 22 LOAD CASE 6 – ANTI-SYMMETRIC LOADING

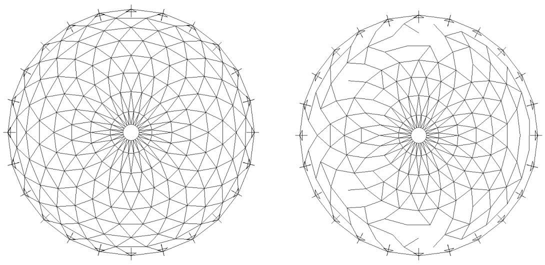


FIG. 23 I-ST STAGE AND V-TH STAGE OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 5

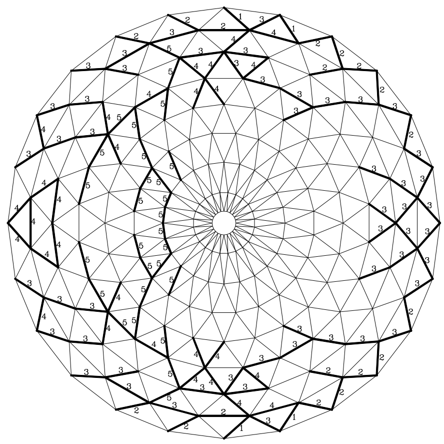
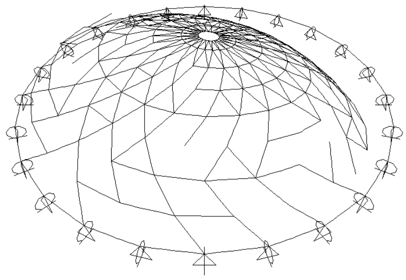


FIG. 24 NUMBER AND SEQUENCE OF BARS REACHING BEARING CAPACITY FOR LOAD CASE 6

The last stage of the exclusion and its corresponding deformed scheme are shown in Fig. 25. Finally under loading of $q_4=4kN/m^2$ total of 131 elements are excluded from the structural system.



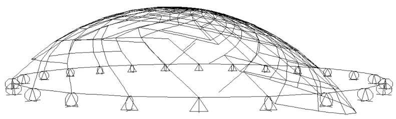


FIG. 25 THE LAST STAGE (FIFTH STAGE) OF EXCLUSION OF ELEMENTS AND ITS CORRESPONDING DEFORMED SCHEME FOR LOAD CASE 6

Conclusions

Studies conducted under six different load cases show different forms (mechanisms) of failure. Apparently, the most unfavorable load configurations are anti-symmetric ones. Limit load of the local symmetric load configuration (third and fourth case) have larger value, thus reflecting a much larger symmetrical stiffness in comparison to the behavior under the anti-symmetric load configuration. In the symmetric local load configurations, elements directly under the load contour are affected. The limit value of the applied load decreases when for anti-symmetric distribution larger area of the dome is covered. This leads to exclusion of large number of meridian elements at the base. Characteristic feature of all load cases is the "domino effect" where the limit load is reached by progressive exclusion of groups of elements at a time.

Based on this research the following conclusions can be formulated:

1) Typical for the dome structure is the elasto-plastic behavior.
2) Progressive collapse is observed when reaching the limit load.
3) The limit load is the lowest for anti-symmetrical load patterns. For local load configurations over small area, local failure under larger limit load is observed. By enlarging the loaded area the value of the limit load decreases.
4) Overloading and subsequent failure is observed for the elements in proximity of the local load stamp.

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